Multirate Rendering of Soft Objects

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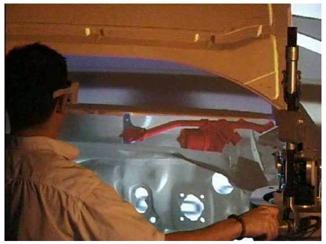
Rigid Tool – Deformable Environment

[Courtesy of Reachin AB]

Virtual Surgery
Scalpel, etc.
Vs.
Soft tissue

Rigid Tool – Deformable Environment



[Courtesy of CEA/Haption]

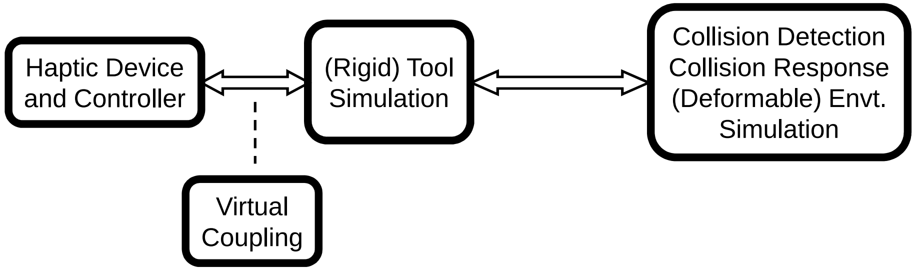
[Courtesy of Boeing]

Virtual Prototyping
Car/aircraft part
Vs.
Wires, flexible structures

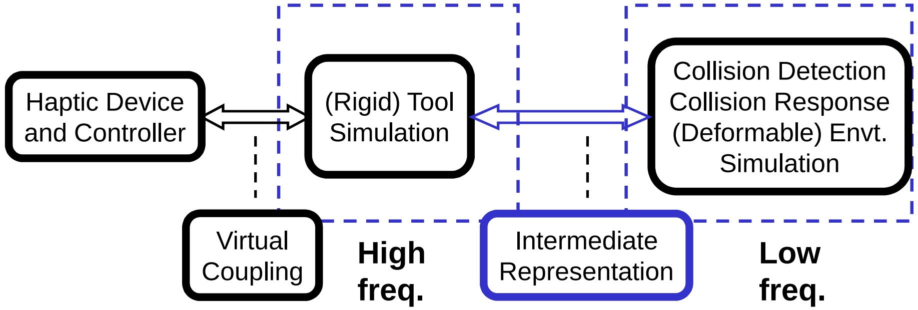
Contents

1. Design of a multirate rendering architecture
2. Constraint-based contact simulation
3. Putting the two together…

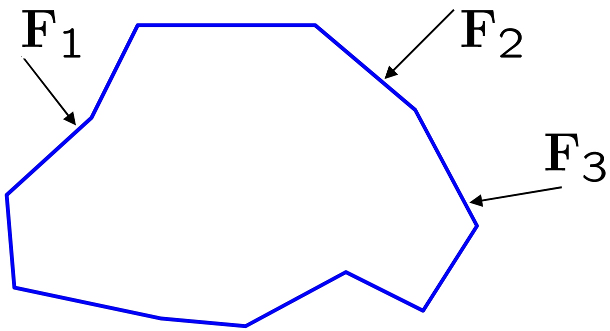
Typical Rendering Algorithm



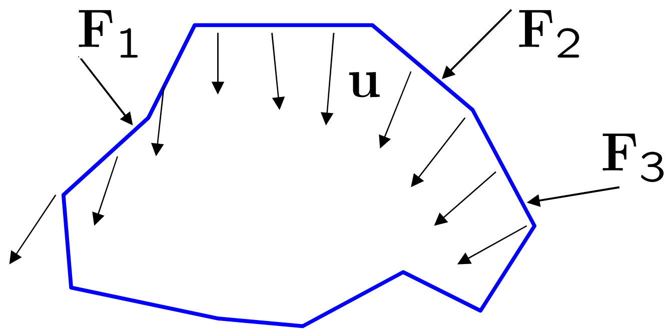
Intermediate Representation



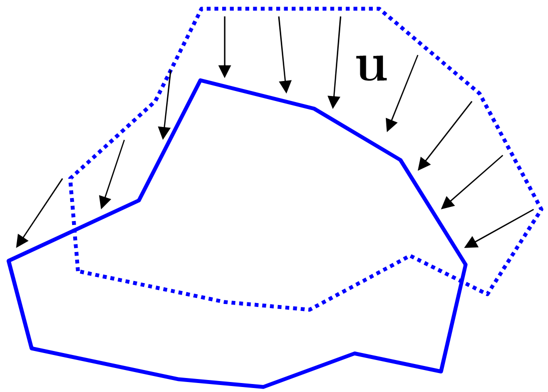
Simulation of Deformable VE



Simulation of Deformable VE



Simulation of Deformable VE



Equations of Motion

$$\mathbf{M}\frac{d\mathbf{v}}{dt} + \mathbf{D}\mathbf{v} + \mathbf{K}\mathbf{u} = \mathbf{f}$$
$$\frac{d\mathbf{u}}{dt} = \mathbf{v}$$

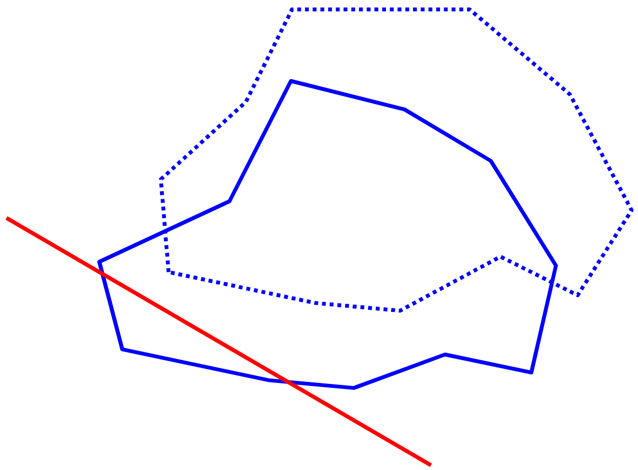
Numerical (Implicit) Integration

$$\mathbf{M}\frac{d\mathbf{v}}{dt} + \mathbf{D}\mathbf{v} + \mathbf{K}\mathbf{u} = \mathbf{f}$$
$$\frac{d\mathbf{u}}{dt} = \mathbf{v}$$

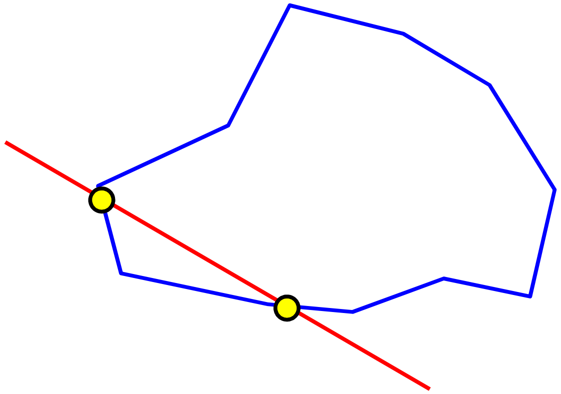
$$\mathbf{A}\mathbf{u}(i+1) = \mathbf{b}$$

$$\mathbf{u}(i+1) \approx \mathbf{u}(i) + \Delta t\,\mathbf{v}(i+1)$$

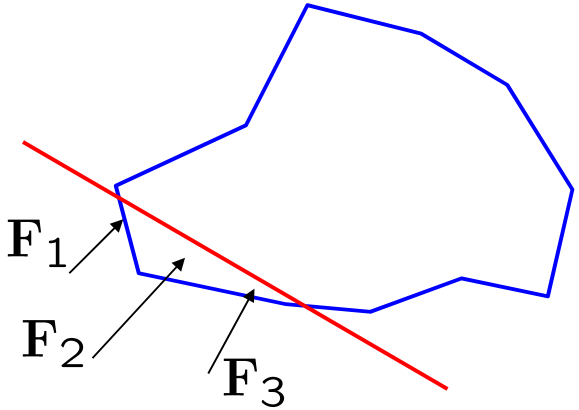
Collision Detection



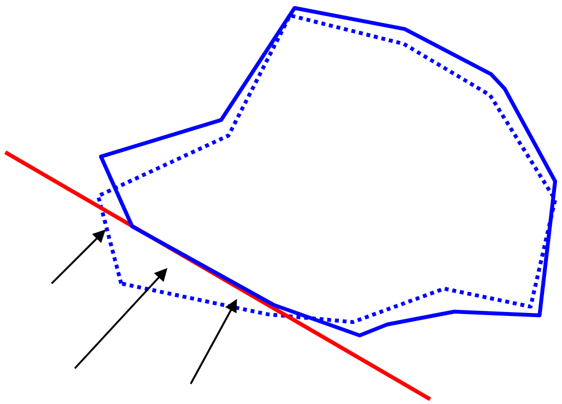
Collision Detection



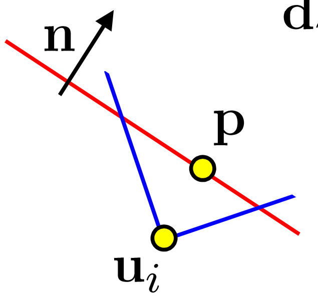
Collision Response



Collision Response

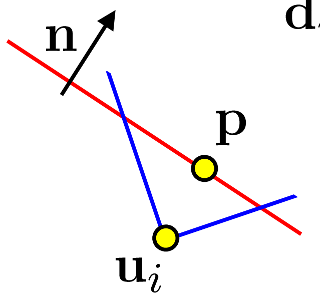


Contact Constraints



$$\mathbf{d}_i = \mathbf{n}^T(\mathbf{u}_i - \mathbf{p}) \geq 0$$

Contact Forces



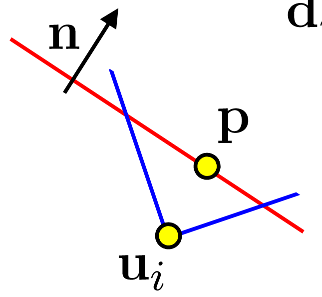
$$\mathbf{d}_i = \mathbf{n}^T(\mathbf{u}_i - \mathbf{p}) \geq 0$$

$$\mathbf{F}_i = \lambda_i \mathbf{n}$$

$$\lambda_i \geq 0$$

Lagrange multipliers

Complementarity



$$\mathbf{d}_i = \mathbf{n}^T(\mathbf{u}_i - \mathbf{p}) \geq 0$$

$$\mathbf{F}_i = \lambda_i \mathbf{n}$$

$$\lambda_i \geq 0$$

$$\mathbf{d}_i \cdot \lambda_i = 0$$

Contact as LCP

$$\mathbf{A}\mathbf{u} = \mathbf{b} + \mathbf{J}^T \lambda$$

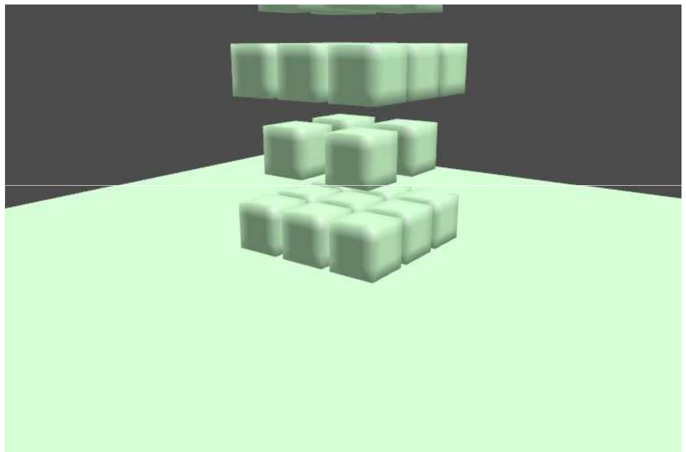
$$\mathbf{J}\mathbf{u} \geq \mathbf{d}$$

$$\lambda \geq 0$$

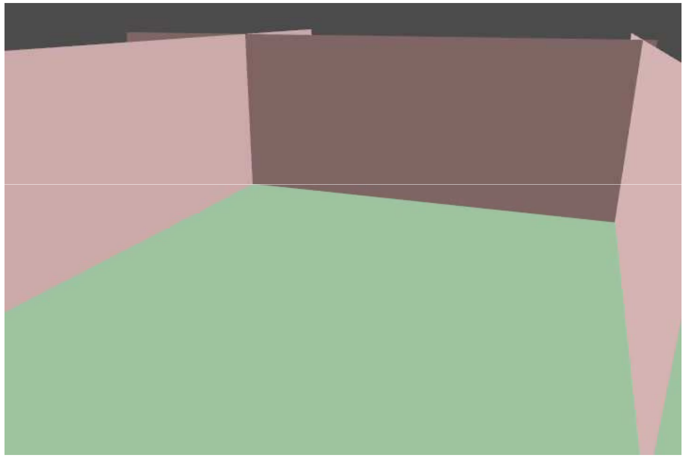
$$\lambda^T(\mathbf{J}\mathbf{u} - \mathbf{d}) = 0$$

LCP = Linear Complementarity Problem

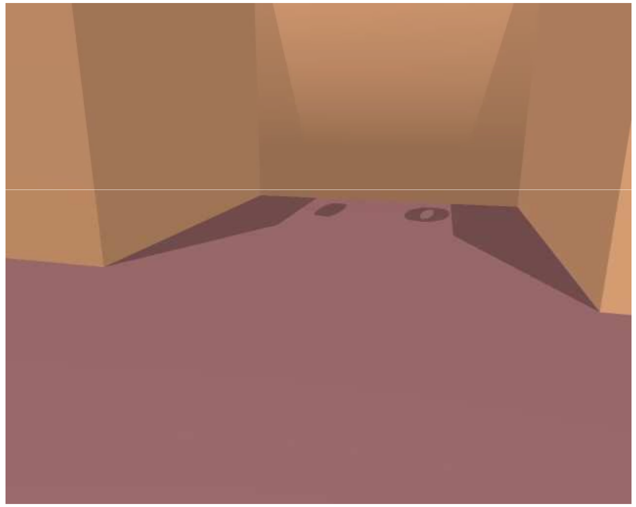
Simulation Examples



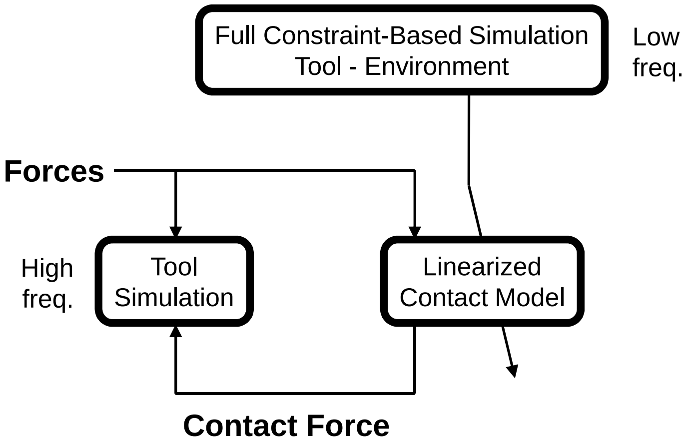
Simulation Examples



Simulation Examples



Linearized Contact Model



Complete System

$$\begin{pmatrix} \mathbf{A}_r & 0 & -\mathbf{J}_r^T \\ 0 & \mathbf{A}_d & -\mathbf{J}_d^T \\ \mathbf{J}_r & \mathbf{J}_d & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u}_r \\ \mathbf{u}_d \\ \lambda \end{pmatrix} = \begin{pmatrix} \mathbf{b}_r \\ \mathbf{b}_d \\ \mathbf{d} \end{pmatrix}$$

$$\mathbf{f}_r = \mathbf{J}_r^T \lambda \approx \mathbf{f}_0 + \frac{\partial \mathbf{f}}{\partial \mathbf{b}_r}(\mathbf{b}_r - \mathbf{b}_0)$$

Linearized Model

Transparent Rendering of Tool Contact
with Compliant Environments (WHC'2007)

Demo: Wednesday 3pm – 4:30pm